

Design of Cantilever Retaining Wall with 4m Height

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Abstract: Retaining structures hold back soil or other loose material where an abrupt change in ground elevation occurs. The retained material or backfill exerts a push on the structure and thus tends to overturn or slide it, or both. The cantilever is the most common type of retaining wall and is used for walls in the range of 3 to 6 m in height. This study presents analyses and design of cantilever retaining wall which is made from an internal stem of steel-reinforced, cast-in-place concrete (often in the shape of an inverted T). In this work a detailed analyses and design for this type of walls which include estimation of primary dimensions of the wall, then these dimensions were checked. The factor of safety against sliding, overturning and bearing were calculated. The shear resistance for the base, the tension stresses in the stem and the tension stresses for the base were checked. Calculation of reinforcement for each part of the wall were done. All analysis and design are based on the ACI code

Keywords: retaining, cantilever, wall, sliding, shear, reinforcement.

I. INTRODUCTION

Retaining walls are usually built to hold back soil mass to retain soil which is unable to stand vertically by themselves. However, retaining walls can also be constructed for aesthetic landscaping purposes. They are also provided to maintain the grounds at two different levels. Retaining walls shall be designed to withstand lateral earth and water pressures, the effects of surcharge loads, the self-weight of the wall. There are many types of retaining walls; following are the different types of retaining walls, based on the shape and the mode of resisting the pressure:

- a. Gravity wall-Masonry or Plain concrete.
- b. Cantilever retaining wall.
- c. Counter fort retaining wall.
- d. Buttress retaining wall.

Cantilever retaining walls are constructed of reinforced concrete. They consist of a relatively thin stem and a base slab. The base is also divided into two parts, the heel and toe. The heel is the part of the base under the backfill. The toe is the other part of the base. The analysis and design of retaining walls includes the following:

- a. Estimation of primary dimensions of the wall, then these dimensions should be checked.
- b. Checking external stability of the walls (sliding of retaining walls, overturning stability and bearing stability)
- c. For reinforced concrete retaining walls main and secondary reinforcement must be calculated.

II. OBJECTIVE OF THE STUDY

This paper shows the analysis and design of the cantilever retaining wall. The design involves two major steps: the first one is the evaluation of the stability of the whole structure under the service loads, which includes the overturning, sliding and bearing failure modes, and the second one is the design of the different components, such as the stem, heel and toe for bending and shear, under the combined factored loads. All analysis and design are based on the ACI code.

1. The design procedure: All design parameters are shown in table 1.

TABLE 1. The design parameters

Parameter	Notation	Value
The height of wall	h	4m
The density of back fill soil	γ_s	17.5 kN/m ³
the angle of internal friction of back fill soil	ϕ	30°
Density of concrete	γ_c	25 kN/m ³
the assumed surcharge load	q	50 kN/m ²
The bearing capacity of soil under the wall	q _a	200 kN/m ²
the angle of internal friction of foundation soil	ϕ	32
The cohesion of foundation soil	C	10 kN/m ²
Compressive strength of concrete	f_c'	21Mpa
The yield stress of steel	f_y	347Mpa

1.1 The dimensions of the retaining wall will be assumed as follow refer to figure1:

a. The width of the wall base

$$B = 0.4H \text{ to } 0.7H = 0.4 * 4 \text{ to } 0.7 * 4$$

B = 2.8m to 1.6m, the width of the base will be assumed as 3.2m

b. The thickness of the stem at the top

$$t = \frac{H}{12} \text{ to } \frac{H}{10}$$

$$t = \frac{H}{10} = \frac{4}{10} = 0.4 \text{ m}$$

$$t = \frac{H}{12} = \frac{4}{12} = 0.3 \text{ m, use 0.3m as thickness of the stem at the top and 0.4m at the base}$$

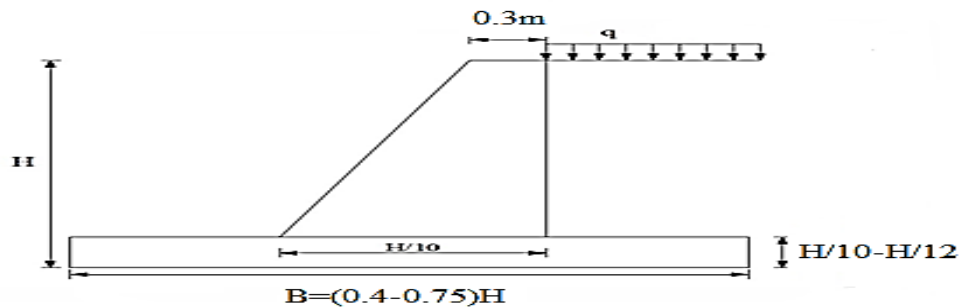


Fig.1 The primary dimension of the wall

c. Length of toe = $\frac{B}{3} = \frac{3.2}{3} = 1.067 \text{ m}$

d. The thickness of the stem at the top will be assumed equal to 0.3m and its thickness at the base = $\frac{H}{10} = 0.4\text{m}$

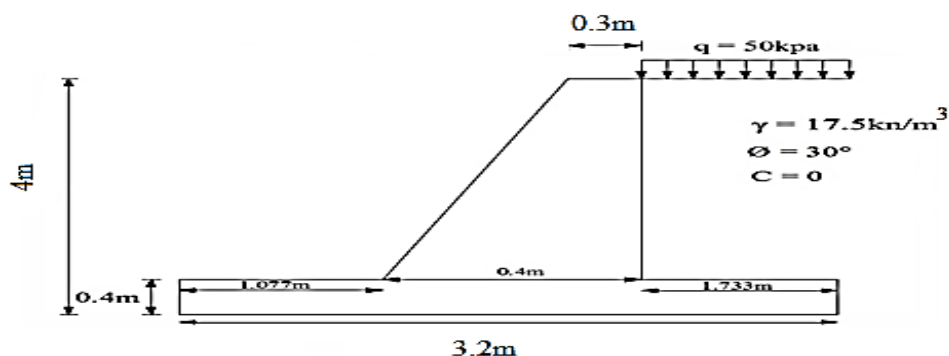


Fig.2 the assumed dimension of the wall

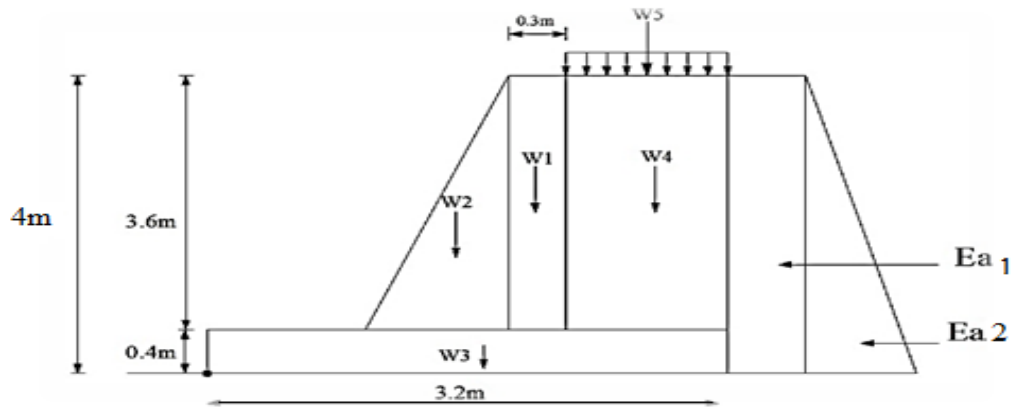


Fig.3. loads and earth pressures acting on the wall

1.2 the loads and earth pressures acting on the wall:

a. The loads:

$$W_1 = 0.3 * 3.6 * 1 * 25 = 27 \text{ KN}$$

$$W_2 = 0.5 * 0.1 * 3.6 * 1 * 25 = 4.5 \text{ KN}$$

$$W_3 = 0.4 * 3.2 * 1 * 25 = 32 \text{ KN}$$

$$W_4 = 3.6 * 1.733 * 1 * 17.5 = 109.179 \text{ KN}$$

$$W_5 = 50 * 1.733 * 1 = 86.65 \text{ KN}$$

The sum of these weights

$$R_v = \sum w = 259.329 \text{ KN}$$

b. The active earth pressures:

$$k_a = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = 0.33$$

$$\sigma_1 = k_a * q$$

$$\sigma_1 = 0.33 * 50 = 16.5 \text{ kpa}$$

$$\sigma_2 = (q + \gamma H) k_a = (50 + (17.5 * 4)) 0.33 = 39.6 \text{ kpa}$$

$$E_{a1} = \sigma_1 * H * 1$$

$$E_{a1} = 16.5 * 4 * 1 = 66 \text{ KN}$$

$$E_{a2} = 0.5(\sigma_2 - \sigma_1) H * 1$$

$$E_{a2} = 0.5 * (39.6 - 16.5) * 4 * 1 = 46.21 \text{ KN}$$

1.3 The stability analysis:

a. Checking the factor of safety against sliding:

$$F.S = \frac{c' * B + R_v \tan \phi}{E_{a1} + E_{a2}}$$

$$c' = c * 0.8 = 8 \text{ kpa}$$

$$F.S = \frac{8 * 3.2 + 259.329 * \tan 32^\circ}{66 + 46.21} = 1.67, F.S > 1.5 \therefore \text{ok, The wall is safe against sliding}$$

b. Checking the factor of safety against overturning:

$$F.S = \frac{\text{resisting moment}}{\text{overturning moment}}$$

The resisting moment and the overturning moment are shown in the following tables.

TABLE 1. The resisting moments

Force KN	Arm m	Moment around point O OKN.m
$W_1 = 27$	1.317	35.559
$W_2 = 4.5$	1.13	5.085
$W_3 = 32$	1.6	1.2
$W_4 = 109.179$	2.3335	254.78
$W_5 = 86.65$	2.3335	202.19
		$M_1 = 548.814 \text{ kN.m}$

TABLE 2. The overturning moments

Force KN	Arm m	Moment around point O
$Ea_1 = 66$	2	132
$Ea_2 = 46.2$	1.33	61.446
		$M_2 = 193.446 \text{ kN.m}$

$$F.S = \frac{M_1}{M_2} = \frac{548.814}{193.446} = 2.8 > 1.5 \text{ the wall is safe against overturning.}$$

c. Checking the pressure under the base of the wall:

$$R_v * x' = M_1 - M_2$$

$$x' = 1.37 \text{ m}$$

$$e = \frac{B}{2} - x'$$

$$e = \frac{3.2}{2} - 1.37 = 0.23 \text{ m}$$

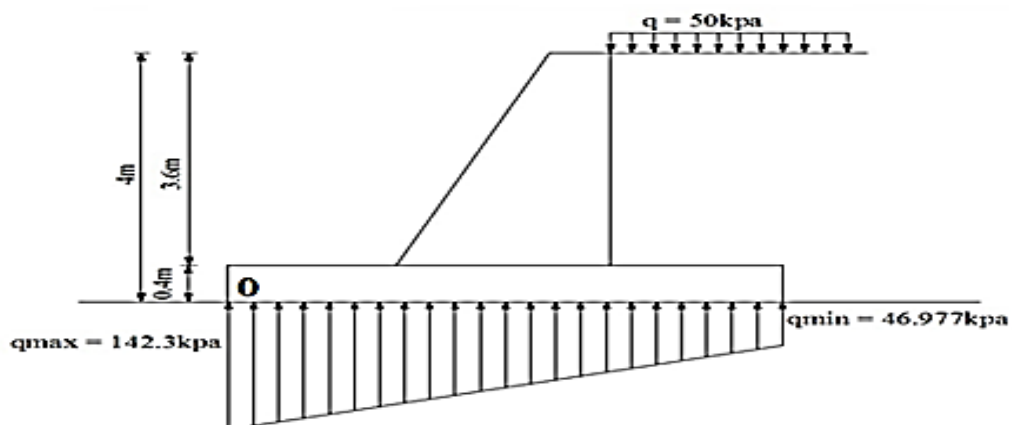


Fig.4. the pressure distribution under the wall

$$B' = B - 2e$$

$$B' = 3.2 - 2 * 0.23 = 2.74 \text{ m}$$

$$q_{\max} = \frac{R_v}{A_f'} \left(1 + \frac{6e}{B'} \right)$$

$$q_{\max} = \frac{259.329}{2.74 * 1} \left(1 + \frac{6 * 0.23}{2.74} \right) = 142.3 \text{ kpa}$$

$$q_{\min} = \frac{Rv}{Af'} \left(1 - \frac{6e}{B'} \right)$$

$$q_{\min} = \frac{259.329}{2.74 * 1} \left(1 - \frac{6 * 0.23}{2.74} \right) = 46.977 \text{ kpa}$$

$$q_a > q_{\max} \therefore \text{ok}$$

1.4 Design of stem reinforcement:

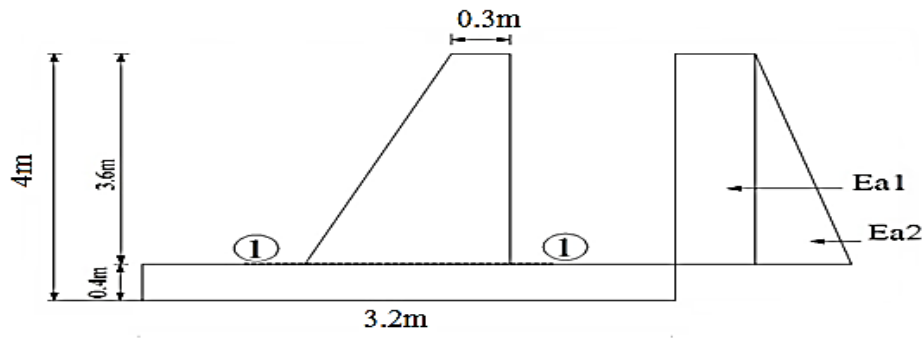


Fig. 5. Shear force on the stem.

The critical section for moment in the stem is at section (1-1) see figure 5 .

Moment in Section (1-1).

$$M = Ea_1 * \frac{h}{2} + Ea_2 * \frac{h}{3}$$

$$M = \sigma_1 h * \frac{h}{2} + ((\sigma_2 - \sigma_1) 0.5 * h * \frac{h}{3})$$

$$M = (16.5 * 3.6 * \frac{3.6}{2}) + ((37.29 - 16.5) 0.5 * 3.6 * \frac{3.6}{3}) = 151.83 \text{ kn.m}$$

$$Mu = M * 1.6 = 151.83 * 1.6 = 242.928 \text{ KN.m}$$

$$Ru = \frac{Mu * 10^6}{0.9 * b * d^2}$$

$$Ru = \frac{242.928 * 10^6}{0.9 * 1000 * (315)^2} = 2.7$$

$$\rho = \frac{0.85 f_c'}{f_y} \left(1 - \sqrt{1 - \frac{2Ru}{0.85 * f_c'}} \right)$$

$$\rho = \frac{0.85 * 21}{347} \left(1 - \sqrt{1 - \frac{2 * 2.7}{0.85 * 21}} \right) = 0.0084$$

$$\rho_{\min} = 0.002$$

$$\rho_{\max} = 0.75 \rho d$$

$$\rho_{\max} = 0.75 \left(0.85 * 0.85 * \frac{f_c'}{f_y} \left(\frac{600}{600 + f_y} \right) \right)$$

$$\rho_{\max} = 0.75 \left(0.85 * 0.85 * \frac{21}{347} \left(\frac{600}{600 + 347} \right) \right) = 0.021$$

$$\rho_{\max} > \rho > \rho_{\min}$$

$$A_s = \rho b d = 0.0084 * 1000 * 315 = 2646 \text{mm}^2$$

Use $\varnothing 20 \therefore A_b = 314 \text{mm}^2$

$$\text{No. of bars} = \frac{A_s}{A_b} = \frac{2646}{314} = 8.4 = 9 \varnothing 20 / \text{m}$$

The Secondary longitudinal reinforcement

$$A_s = \frac{t_1 + t_2}{2} * 1000 * 0.002$$

$$A_s = \frac{300 + 400}{2} * 1000 * 0.002$$

Use $\varnothing 10$, $A_b = 78.5 \text{mm}^2$

$$\text{No. of bars} = \frac{A_s}{A_b} = 11 \varnothing 10 / \text{m}$$

Transverse temperature and shrinkage reinforcement

$$A_{s_{\min}} = \frac{t_1 + t_2}{2} * h * 0.002$$

$$A_{s_{\min}} = \frac{300 + 400}{2} * 3600 * 0.002 = 2520 \text{mm}^2$$

Use $\varnothing 10$, $A_b = 79 \text{mm}^2$

$$\text{No. of bars} = \frac{2520}{79} = 32.08$$

Use 33 \varnothing 10

One third for inside face of the wall and two – third for outside face of the wall.

1.6 Design of the base of the wall:

a. The shear force for the base

$$q_1 = q * 1.6$$

$$q_1 = 50 * 1.6 = 80 \text{ kpa}$$

$$q_2 = \gamma_{\text{soil}} * h * 1.2 = 17.5 * 3.6 * 1.2 = 75.6 \text{ kpa}$$

$$q_3 = \gamma_c * t * 1.2 = 25 * 0.4 * 1.2 = 12 \text{ kpa}$$

$$\sum q = 169.6 \text{ kpa}$$

$$qu_1 = q_{\max} * 1.6 = 142.3 * 1.6 = 227.68 \text{ kpa}$$

$$qu_2 = q_{\min} * 1.6 = 46.977 * 1.6 = 75.1632 \text{ kpa}$$

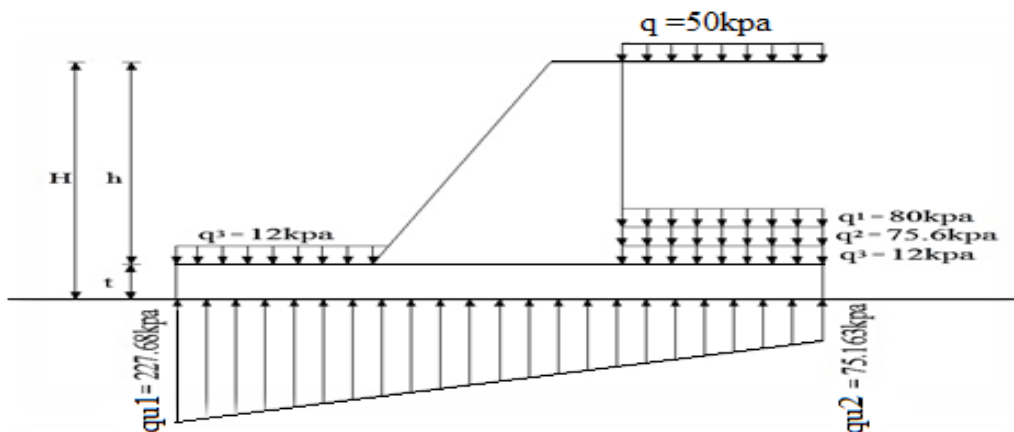


Fig.6. the pressures on the base

The ultimate shear force on the base:

$$V_u = \frac{q' + qd}{2} * L = \frac{94.44 + 26.85}{2} * 1.418 = 85.99 \text{ KN}$$

$$d = 400 - 75 - \frac{20}{2} = 315 \text{ mm}$$

The shear resistance:

$$V_c = v_c * b * d$$

$$v_c = 0.17 * 0.85 * \sqrt{21} * 1000 * 1 * 0.315 = 208.5 \text{ KN}$$

$V_c > V_u$: the thickness of the base is ok

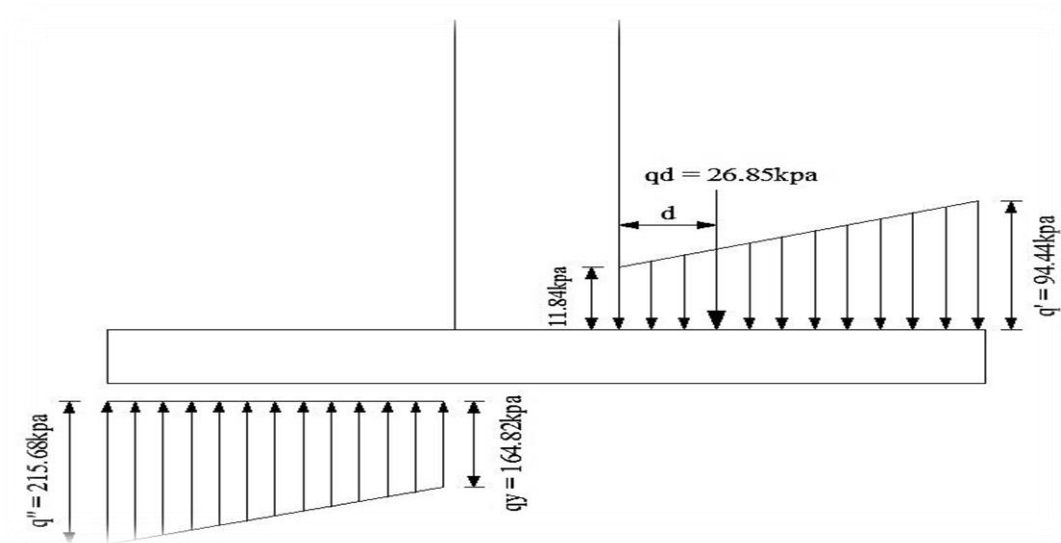


Fig. 7 the net pressure on the base

b. The reinforcement of the heel:

Dived the pressure which acts on the heel to rectangular and triangular shape and take the moment of these shapes around the face of the stem see figure 9,

$$M = (11.84 * 1.733) * (\frac{1.733}{2}) + (0.5 * 1.733 * 82.6) * (\frac{2}{3} * 1.733)$$

$$M = 17.77 + 82.69 = 100.46 \text{ KN.m}$$

$$M_u = 100.46 * 1.6 = 160.736 \text{ KN.m}$$

$$R_u = \frac{M_u * 10^6}{0.9 * b * d^2} = \frac{160.736 * 10^6}{0.9 * 1000 * 315^2} = 1.79$$

$$\rho = \frac{0.85 f_c'}{f_y} \left(1 - \sqrt{1 - \frac{2R_u}{0.85 * f_c'}} \right)$$

$$\rho = \frac{0.85 * 21}{347} \left(1 - \sqrt{1 - \frac{2 * 4.2}{0.85 * 21}} \right) = 0.0054$$

$$\rho_{min} = 0.002$$

$$\rho_{max} > \rho > \rho_{min}$$

$$A_s = \rho * b * d = 0.0054 * 1000 * 315 = 1701 \text{ mm}^2$$

$$\text{Use } \phi 20 \therefore A_b = 314 \text{ mm}^2$$

$$\text{No .of bars} = \frac{A_s}{A_b} = \frac{1701}{314} = 5.41$$

Use 6Ø 20/ m top.

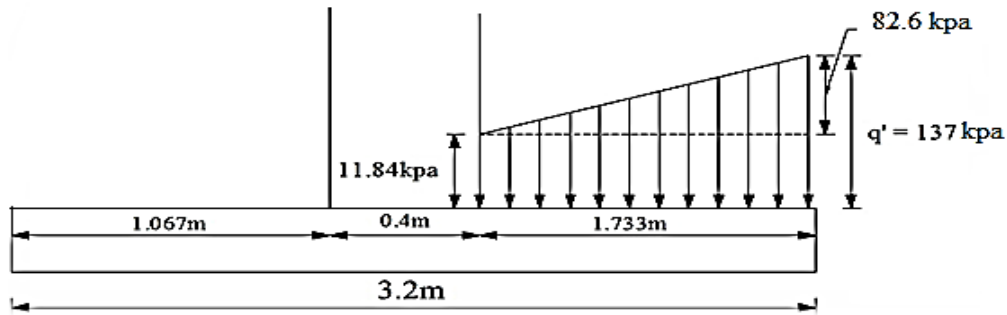


Fig. 8 The pressure on the heel

c. The shear force on the toe:

$$V_u = \frac{q'' + qd}{2} * L$$

$q'' = 215.68 \text{ kpa}$, $qd = 179.88 \text{ kpa}$ and $L = 0.752 \text{ m}$

$$V_u = \frac{215.68 + 179.88}{2} * 0.752 = 148.731 \text{ kN}$$

$$V_c = 208.5 \text{ kN}$$

$V_c > V_u$: t is ok

d. the reinforcement of the toe:

$$M = [(164.82 * 1.067) * \frac{1.067}{2}] + [(0.5 * 1.067 * 50.86) * (\frac{2}{3} * 1.067)] = 112.5 \text{ KN.m}$$

$$M_u = M * 1.6 = 180 \text{ KN.m}$$

$$R_u = \frac{M_u * 10^6}{0.9 * b * d^2} = \frac{180 * 10^6}{0.9 * 1000 * 315^2} = 2.01$$

$$\rho = \frac{0.85 f_c'}{f_y} \left(1 - \sqrt{1 - \frac{2 R_u}{0.85 * f_c'}} \right)$$

$$\rho = \frac{0.85 * 21}{347} \left(1 - \sqrt{1 - \frac{2 * 1.85}{0.85 * 21}} \right) = 0.0061 > \rho_{\min}$$

$$A_s = \rho b d = 0.0061 * 1000 * 315 = 1921.5 \text{ mm}^2$$

Use Ø 20 ∴ $A_b = 314 \text{ mm}^2$

7 Ø 20 /m bottom

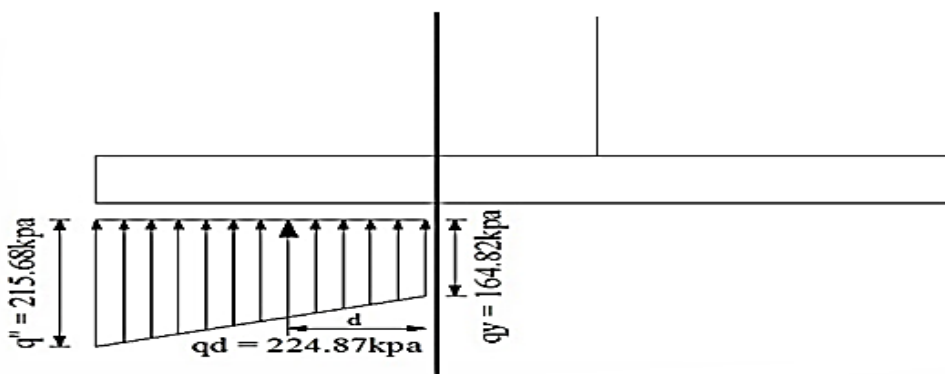


Fig. 9 Pressure on the toe

The secondary reinforcement .

$$A_{s_{min}} = 0.002 \times b \times t = 0.002 \times 3200 \times 400 = 2560/2 = 1280 \text{ mm}^2$$

Use $\phi 10$, $A_b = 79 \text{ mm}^2$

$$\text{The spacing } S = \frac{79}{1280} * 1000 = 61.7 \text{ mm}$$

$\phi 10@60\text{mm}$ top and bottom.

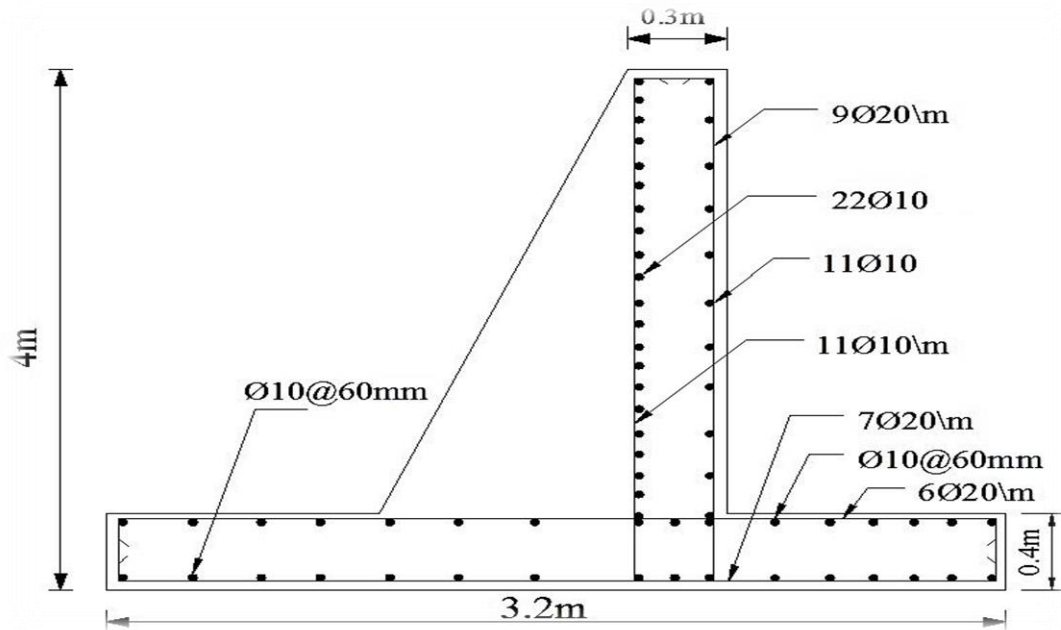


Fig.10 The reinforcement of the cantilever R.W

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